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PH.D. Qualifying Exam in Analysis

for IAN WALTON - January 3, 1975

1:00 - 4:00 p.m.

1. Is there a bounded, non-constant complex analytic function defined on all of the complex plane? If so, give an example. If not, prove that there is none.

2. Prove that ℓ_1 and ℓ_2 are not isomorphic topological vector spaces.

3. Evaluate $\int_0^\infty \frac{\log x}{1 + x^2} \, dx$

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4. Let $\mathcal{C}$ be the Cantor set on $[0,1]$. Suppose that $f(x) = 0$ on $\mathcal{C}$ and $f(x) = \frac{1}{2^n}$ on each of the intervals of the complement of $\mathcal{C}$ of length 3^{-n}. Show that $f \in L_1(0,1)$ and evaluate $\int_0^1 f(x)\,dx$. Is f Riemann integrable?

5. For what real values of α is $x^\alpha \sin \frac{1}{x}$ of bounded variation in $(0,1)$?

6. Determine $\min_{a,b} \int_{-\pi}^{\pi} |x - a\cos x - b\sin x|^2\,dx$

7. Suppose that f is a nonnegative measurable function in $(0,\infty)$ such that $e^{-x/n} f(x) \in L_1(0,\infty)$ for each positive integer n. Prove that if $\lim\limits_{n\to\infty} \int_0^\infty e^{-x/n} f(x)\,dx = \alpha$, then $f \in L_1(0,\infty)$ and $\int_0^\infty f(x)\,dx = \alpha$.