1. Show that the alternating group of degree 5, $\mathrm{Alt}(5)$, has no subgroup of order 20.

2. Show that every group of order 143 is cyclic.

3. Show that every non-zero ideal of the localization $\mathbb{Z}_{(p)}$ of $\mathbb{Z}$ at a prime p is of the form (p^n) for some $n \geq 0$.

4. Let T be the $\mathbb{R}$-linear map on $V = \mathbb{R}^2$ given by the multiplication with the matrix

$$\begin{pmatrix} 0 & -1 \\ 4 & 2 \end{pmatrix}.$$

Compute the characteristic polynomial of $T \otimes_{\mathbb{R}} T$ acting on $V \otimes_{\mathbb{R}} V$.

5. Let N be the subgroup of the free abelian group $M = \mathbb{Z}^3$ generated by the following 3 elements:

$$x = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \ y = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \ \text{and} \ z = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix},$$

and let $Q := M/N$.

   (a) Compute the rank of Q.

   (b) Compute the invariant factors of Q.

6. Let p be a prime number, $F = \mathbb{F}_p$ the field with p elements, and $V = F^4$ the standard 4-dimensional F-vector space. Count the number of 2-dimensional F-linear subspaces in V.

7. (a) Determine the Galois group of $x^8 - 11$ over $\mathbb{Q}$.

   (b) Determine the Galois group of $x^8 - 11$ over $\mathbb{Q}(i)$.

8. Let $K \supset E \supset F$ be fields. Prove or disprove:

   (a) If $K \supset F$ is Galois then $E \supset F$ is Galois,

   (b) If $K \supset F$ then $K \supset E$ is Galois,

   (c) If both $K \supset E$ and $E \supset F$ are Galois then $K \supset F$ is Galois.

9. Determine whether or not $x^5 - 4x + 2$ is solvable by radicals.