1. Show that every group of order 2016 has a subgroup of order 28.

2. Determine the minimal \( n \in \mathbb{N} \) such that the quaternion group \( Q_8 \) of order 8 is isomorphic to a subgroup of the symmetric group \( \text{Sym}(n) \). (Hint: Consider the resulting action of \( Q_8 \) on \( \{1, 2, \ldots, n\} \) and use that every non-trivial subgroup of \( Q_8 \) contains the unique element of order 2.)

3. Let \( p \) be a prime. Consider the subring \( R \) of \( \mathbb{Q} \) consisting of all fractions \( a/b \) with \( a, b \in \mathbb{Z} \) such that \( b \) is not divisible by \( p \). Show that every non-zero ideal of \( R \) is of the form \( p^kR \) for some integer \( k \geq 0 \).

4. Suppose that \( A \) is a \( 5 \times 5 \) complex matrix satisfying
   \[
   A^4 + 4A^2 = 4A^3.
   \]
   (a) What are the possible eigenvalues of \( A \)?
   (b) Determine all possible Jordan canonical forms of \( A \).

5. Prove or disprove: For any integer \( n \geq 1 \), any \( n \times n \) matrix \( B \) over an algebraically closed field \( k \) is diagonalisable, if \( B^r = I_n \) for some integer \( r \geq 1 \).

6. Prove or disprove: Any vector \( \vec{v} \in \mathbb{R}^3 \) is the cross product \( \vec{u} \times \vec{w} \) for some vectors \( \vec{u}, \vec{w} \) in \( \mathbb{R}^3 \).

7. Let \( R \) be a commutative ring with identity. For each of the following, either prove the statement or give a counterexample:
   (a) The tensor product of two injective \( R \)-modules is injective.
   (b) The tensor product of two flat \( R \)-modules is flat.
   (Note: you should prove or disprove these statements from scratch, not apply theorems that render the conclusions trivial.)

8. Let \( \mathbb{C}(x) \) denote the field of rational functions over \( \mathbb{C} \) in an indeterminate \( x \). For each \( a \in \mathbb{C} \), let
   \[
   \sigma_a : \mathbb{C}(x) \longrightarrow \mathbb{C}(x)
   \]
   denote the field automorphism that substitutes \( x \) by \( x + a \). Let \( G \) denote the group of automorphisms \( \{ \sigma_a : a \in \mathbb{C} \} \). Determine with proof the subfield \( \mathbb{C}(x)^G \), the subfield of \( \mathbb{C}(x) \) fixed by \( G \).

9. Let \( \lambda \) be an eigenvalue of a linear transformation \( T \) of a finite dimensional vector space \( V \) over an algebraically closed field \( F \). Let \( r_k = \dim_F (T - \lambda)^k V \), i.e. \( r_k \) is the rank of the linear transformation \( (T - \lambda)^k \) on \( V \). Prove that for any \( k \geq 1 \), the number of Jordan blocks of \( T \) corresponding to \( \lambda \) of size \( k \) is \( r_{k-1} - 2r_k + r_{k+1} \).