Analysis Prelim Spring 2001

Print Your Name:

All questions should be answered on this exam using the backs of the sheets if necessary. The exam has 9 pages.

Show all your work and justify all claims when a proof is required. Good Luck!!

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Total	/100

1. (10 points) Let $K \subset \mathbb{R}$ be a compact set of measure zero. Prove that for any $\epsilon > 0$ there exists a finite collection of intervals $I_1, \ldots, I_n$ covering K such that the total length of the intervals is less than ϵ, i.e., $\sum |I_i| < \epsilon$.

2. (10 points) Prove that l_∞ is not separable.

3. (15 points) Assume that f is uniformly continuous on $[0, \infty)$ and $f \in L^p([0, \infty))$, where $1 \leq p < \infty$. Prove that $f(x) \to 0$ as $x \to \infty$.

4. (15 points) Evaluate

$$\int_0^\infty \frac{x \sin x}{x^2 + a^2}\, dx.$$

Remark: You may use Jordan's lemma without proof.

5. (10 points) Let f be an entire function such that $|f(z)| \leq M(1+|z|)^n$ for some n and some $M \geq 0$. Prove that f is a polynomial of degree less than or equal to n.

6. (10 points) Let $P(z) = z^7 + z^3 + 2z^2 + z$. How many zeros (counting with multiplicity) does $P(z)$ have in the disc or radius 2 centered at 0.

7. (15 points) Prove that the Fourier transform maps $L^1(\mathbb{R})$ to $C^0_0(\mathbb{R})$, where $C^0_0(\mathbb{R})$ denotes the space of continuous functions which tend to zero at infinity.

8. $A\colon l_p \to l_p$ be given by the formula $A(\{x_n\}) = \{a_n x_n\}$, where $\{a_n\}$ be a bounded sequence.

(a) (5 points) Prove that A is bounded.

(b) (10 points) Assume in addition that $a_n \to 0$. Prove that A is compact. Hint: the limit of a sequence of compact operators is compact.