1. Suppose that \( \{f_n\}_{n=1}^{\infty} \) is a sequence of monotonically increasing real-valued continuous functions on \([0, 1]\). Assume that \( f_n \) converges pointwise to a continuous function \( f \) on \([0, 1]\). Show that the family \( \{f_n\}_{n=1}^{\infty} \) is uniformly equi-continuous on \([0, 1]\).

2. Suppose that a sequence of real numbers \( x_n \) converges to a real number \( x_0 \). Show that
\[
\lim_{n \to \infty} \frac{x_1 + x_2 + \cdots + x_n}{n} = x.
\]

3. Let \( f : [0, 1]^2 \to \mathbb{R} \) be a function such that \( f(x, y) \) is Lebesgue integrable in \( x \) for each fixed \( y \), and differentiable in \( y \) for each fixed \( x \). Assume there is a Lebesgue integrable function \( g(x) \) such that \( \left| \frac{\partial f}{\partial y}(x, y) \right| \leq g(x) \) for every \( x, y \). Show that \( \psi(y) := \int_0^1 f(x, y) \, dx \) can be differentiated under the integral sign.

4. Let \( \mu \) and \( \nu \) be finite nonnegative measures on a measure space \((X, \mathcal{Q})\), such that \( \mu \ll \nu \). Let \( \frac{d\nu}{d(\mu+\nu)} \) stand for the Radon-Nykodim derivative of \( \nu \) with respect to \( \mu + \nu \). Show that
\[
0 < \frac{d\nu}{d(\mu+\nu)} < 1 \quad \text{[\( \mu \)]-a.e.}
\]

5. Suppose that \( X \) is a normed vector space and that \( W \) is a closed subspace in \( X \). Let \( x_0 \in X \). Assume that \( \phi(x_0) = 0 \) for all \( \phi \in X^* \) with \( N(\phi) \supseteq W \). Show that \( x_0 \in W \).

6. Suppose that \( C(\Omega) \) is the space of all continuous functions from a bounded connected and open domain \( \Omega \) in the Euclidean space \( \mathbb{R}^n \). Define a vector topology on the space \( C(\Omega) \) such that it becomes a Fréchet space. (Prove that it becomes a Fréchet space.)

7. State the Casorati-Weierstrass Theorem. Using it show that the only bi-holomorphic maps of \( \mathbb{C} \) to itself are mappings of the form \( f(z) = Az + B \).

8. Let \( \Omega \) be a connected open domain in \( \mathbb{C} \) and \( f \) be a holomorphic function on \( \Omega \) which does not vanish identically. Show that the zeros of \( f \) are isolated.