CHOOSE A 'CODE' (not part of your soc sec number) for yourself. Put it on each page. Attach extra pages as necc.

1. For what values of the constant c is the level set $x^2 + y^2 - zw = c$ an embedded submanifold of $\mathbb{R}^4$?

2. Consider the vector field $X = \Sigma x^i \frac{\partial}{\partial x^i}$ in $I\!R^n$.
a) Compute the flow of X

b) Find constants k and one-forms α on $I\!R^n$ such that $L_X \alpha = k\alpha$, where L_X denotes the Lie derivative along the vector field X.

c) Prove that the only globally defined one-form α invariant under X's flow is the trivial one-form $\alpha = 0$.

The next three questions are true or false questions. If true, give a proof. If false, provide a counterexample. In these questions X and Y are smooth manifolds and $F : X \to Y$ is a submersion.

3. True or False? If X is connected then Y is connected.

4. True or False? If Y is connected then X is connected.

5. True or False? If both X and Y are compact and connected, and if Y is simply connected then X is simply-connected.

6. . (a) Describe a coordinate chart for the Grassmannian of 2-planes in $I\!R^4$.

b) The group $SO(4)$ of rotations of $I\!R^4$ acts on this Grassmannian.. Show that this action is transitive.

c) Compute the isotropy group for the action at the plane $x_3 = x_4 = 0$ where x_1, x_2, x_3, x_4 are the standard coordinates for $I\!R^4$.

7. Consider the one-form $\theta = dz - y^2 dx$ on $I\!R^3$.

a) Find a frame V_1, V_2, V_3 of vector fields for $I\!R^3$ such that $\theta(V_3) = 1$ while $\theta(V_1) = \theta(V_2) = 0$ and which agrees with the standard coordinate frame $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ at the origin, expressing the elements of your frame in terms of the coordinate standard frame.

b) An infinitesimal symmetry of θ is a smooth vector field X such that $L_X\theta = f\theta$ for some smooth function $f(x, y, z)$, where L_X denotes the Lie derivative along the vector field X. By expanding X in terms of the frame as $X = h_1V_1 + h_2V_2 + h_3V_3$, show that if X is an infinitesimal symmetry of X then $h_2 = 0$ and $h_3 = h_3(x, z)$ does not depend on y.

c) Show that X must be tangent to the surface $y = 0$.