CHOOSE A ‘CODE’ (not part of your soc sec number) for yourself. Put it on each page. Attach extra pages as necc.

1. For what values of the constant $c$ is the level set $x^2 + y^2 - zw = c$ an embedded submanifold of $\mathbb{R}^4$?
2. Consider the vector field \( X = \Sigma x^i \frac{\partial}{\partial x^i} \) in \( \mathbb{R}^n \).
a) Compute the flow of \( X \)

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\text{b) Find constants } k \text{ and one-forms } \alpha \text{ on } \mathbb{R}^n \text{ such that } L_X \alpha = k \alpha, \text{ where } L_X \text{ denotes the Lie derivative along the vector field } X.
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\text{c) Prove that the only globally defined one-form } \alpha \text{ invariant under } X \text{'s flow is the trivial one-form } \alpha = 0.
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The next three questions are true or false questions. If true, give a proof. If false, provide a counterexample. In these questions X and Y are smooth manifolds and $F : X \to Y$ is a submersion.

3. True or False? If $X$ is connected then $Y$ is connected.

4. True or False? If $Y$ is connected then $X$ is connected.

5. True or False? If both $X$ and $Y$ are compact and connected, and if $Y$ is simply connected then $X$ is simply-connected.
6. (a) Describe a coordinate chart for the Grassmannian of 2-planes in $\mathbb{R}^4$.

b) The group $SO(4)$ of rotations of $\mathbb{R}^4$ acts on this Grassmannian. Show that this action is transitive.

c) Compute the isotropy group for the action at the plane $x_3 = x_4 = 0$ where $x_1, x_2, x_3, x_4$ are the standard coordinates for $\mathbb{R}^4$. 
7. Consider the one-form \( \theta = dz - y^2 dx \) on \( \mathbb{R}^3 \).

a) Find a frame \( V_1, V_2, V_3 \) of vector fields for \( \mathbb{R}^3 \) such that \( \theta(V_3) = 1 \) while \( \theta(V_1) = \theta(V_2) = 0 \) and which agrees with the standard coordinate frame \( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \) at the origin, expressing the elements of your frame in terms of the coordinate standard frame.

b) An infinitesimal symmetry of \( \theta \) is a smooth vector field \( X \) such that \( L_X \theta = f \theta \) for some smooth function \( f(x, y, z) \), where \( L_X \) denotes the Lie derivative along the vector field \( X \). By expanding \( X \) in terms of the frame as \( X = h_1 V_1 + h_2 V_2 + h_3 V_3 \), show that if \( X \) is an infinitesimal symmetry of \( X \) then \( h_2 = 0 \) and \( h_3 = h_3(x, z) \) does not depend on \( y \).

c) Show that \( X \) must be tangent to the surface \( y = 0 \).