

Geometry and Topology Prelim, UCSC, Winter 2016

1. Consider the vector fields

$$v = x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}$$

and

$$w = x^m \frac{\partial}{\partial y} + y^m \frac{\partial}{\partial x}.$$

on \mathbb{R}^2 , where $m \geq 2$

- (a) Are these vector fields complete?
- (b) Find the flow of v .
- (c) Find the bracket $[v, w]$.

2. Which of the following manifolds are orientable and which are not: $\mathbb{R}P^n$, $\mathbb{C}P^n$, $SO(n)$ for $n = 1, 2, \dots$. Justify your answer.

3. Prove that a closed 1-form on TS^1 is exact if and only if its integral over the zero section is zero.

4. Show that the set X of orthogonal $n \times n$ matrices with determinant equal to -1 is a manifold. Is the tangent bundle TX trivial?

5. Let X be a quotient space of an annulus obtained by identifying points which are 90 degrees apart on the outer circle, and identifying points on the inner circle which are 120 degrees apart. Find $\pi_1(X)$.

6. Let $\chi(X)$ be the Euler characteristic of a finite cell complex X . Let A, B be its subcomplexes such that $X = A \cup B$. Show that

$$\chi(X) = \chi(A) + \chi(B) - \chi(A \cap B).$$

7. Find the Gaussian curvature (aka the scalar curvature) of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

at the point $(0, 0, c)$.