Characterization of minimal models (an introduction)
Upon retirement of Professor Geoffrey Mason

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On this occasion I talk on a couple of properties which characterizes a family of simple Virasoro vertex operator algebras called **the minimal models**[1].

Let V be a simple vertex operator algebra **of CFT type** with the space of characters of simple modules whose dimension is between 2 and 6. Since the case that its dimension is between 4 and 6 is relatively complicated, I decide to explain dimension 2 and 3 cases so that the audiences can roughly understand our method. Even dimension 3 case an interesting problem appears that if C_2-cofinite rational vertex operator algebras with the central charge $236/7 \approx 33.7$ and $164/5 \approx 32.8$ exist.

Suppose that V is **rational** and **C_2-cofinite**. Then the set of characters (one-point functions) forms a vector-valued modular function (modular form of weight 0) which G. Mason have studied. There exists a mild condition that the space which is linearly generated by a basis of the space of characters with the dimension n coincides with the space of solutions of a **modular linear differential equation** of order n, which was also found by G. Mason (and C. Marks). Using this equation, we can reduce the characterization problem of a family of simple vertex operator algebras to a Diophantus equation of a *formal*[2] **central charge** c, whose degree is at least n (this degree has order n^2 when n goes to the infinity). By using a computer we found all integral solutions of the Diophantus equation for n between 2 and 6), which lead us the complete characterization of the minimal models for $2 \leq n \leq 6$. However, to solve the Diophantus equation for $n = 6$, our computer took a whole day. Therefore, we cannot expect that this approach work for $n > 6$.

It is proved by C. Dong, X. Lin and R. Ng that any characters of a rational C_2-cofinite self-dual vertex operator algebra is a modular function with level N (which is explicitly determined by an effective central charge of V). Based on this fact we find a way to solve exact solutions by using modular forms with weights k_1 and k_2 such that $k_1 - k_2 = \tilde{c}/2$, where $\tilde{c}$ is an effective central charge of the theory. The weak point of this method is that we must know a basis of the space of modular forms of weight k_i (which are mostly rational) on a congruence subgroup of $SL(2, \mathbb{Z})$.

Finally, we mention that we have modular functions, quasimodular forms, mixed mock modular forms as solutions of the modular linear differential equations which appear in this talk.

This talk must be self-contained, which is why I use the beamer. Therefore, prior knowledge on the theory of vertex operator algebras and the bold faced terminologies are not required.

[1] All bold face terminologies are explained in the lecture.

[2] Since at this moment it is not clear if the solutions are central charges, we use the adjective "formal".